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Acoustic power and heat fluxes in the thermoacoustic effect due to a travelling plane wave

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Abstract—The thermoacoustic effect in a single plate due to a travelling plane wave is analyzed with respect to the conditions for the refrigerator or prime mover cycle to occur. The relation between the conduction of heat along a normal to the plate and the absorbed or generated acoustic power is also shown.

1. INTRODUCTION

In a previous work [1] we analyzed the thermoacoustic effect regarding heat flux and acoustic power generated or absorbed for the case of a single plate in a stationary plane wave field. In this work a similar kind of analysis is made, also for a single plate, but now immersed in a travelling plane wave field. The considerations with respect to the coordinate system, temperature gradient, variables, etc., are the same and follow, as before, Rott's theory [2].

2. VELOCITY AND TEMPERATURE OSCILLATIONS IN THE FLUID

The spatial dependence of the fluid velocity parallel to the surface of the plate is given by ref. [2], equation (2.2)

$$u_1 = \frac{i}{\omega\rho_m}\frac{\partial p_1}{\partial x}(1-e^{-(1+i)y/\delta_\nu}) \tag{1}$$

where ω is the angular frequency of the sound wave, p_1 is the spatial dependence of the pressure oscillations, ρ_m is the mean density of the fluid, $\delta_\nu=(2\nu/\omega)^{1/2}$ its viscous penetration depth and ν its kinematic viscosity. This equation is valid if the displacement of the fluid is much smaller than the length (Δx) of the plate ($|p_1|/(\rho_m\omega c)\ll\Delta x$, where c is the sound speed). An expression for T_1 is given by ref. [1], equation (3):

$$T_1 = \frac{T_m\beta}{\rho_m c_p}p_1 - \frac{1}{\rho_m\omega^2}\frac{dp_1}{dx}\nabla T_m\left(1-\frac{\sigma}{\sigma-1}e^{-(1+i)y/\delta_\nu}\right) - \left(\frac{T_m\beta}{\rho_m c_p}p_1+\frac{1+\varepsilon_s\sqrt{\sigma}}{(\sigma-1)\rho_m\omega^2}\frac{dp_1}{dx}\nabla T_m\right)\frac{e^{-(1+i)y/\delta_\kappa}}{1+\varepsilon_s} \tag{2}$$

with

$$\varepsilon_s = \frac{\delta_\kappa\rho_m c_p}{\delta_s\rho_s c_s} \quad (\text{supposing } \delta_s \ll l), \tag{3}$$

where T_m is the mean temperature; $\delta_s=(2\kappa_s/\omega)^{1/2}$, κ_s, and c_s are, respectively, the thermal penetration depth, the thermal diffusivity, and the isobaric heat capacity per unit mass of the plate; $\delta_\kappa=(2\kappa/\omega)^{1/2}$, κ, and c_p, are the corresponding parameters of the fluid; σ is the Prandtl number of the fluid, β its thermal expansion coefficient; ρ_s is the density of the plate and l its thickness. This expression is valid for $\sigma<1$. An equivalent equation for T_1 was given by Rott ([2] pp. 140–141), who also obtained an expression for the case that $\sigma=1$. When y tends to infinity, equations (1) and (2) are reduced to the well known expressions for sound waves in free space where a mean gradient of temperature exists.

3. ANALYSIS AND RESULTS

To keep things manageable, it is assumed that the wavelength ($2\pi c/\omega$) is much longer than the dimensions of the plate ('short engine' approximation) and that the acoustic pressure is much smaller than the mean pressure ($|p_1|\ll p_m$). Note that the amplitude of the oscillations in the fluid is much smaller than the wavelength, therefore the existence of a Δx that satisfies both equations (1) and (2) is guaranteed, thus avoiding the apparent contradiction between a 'short plate' approximation as was done in equation (2) and equation (1) which corresponds to a 'long plate' approximation. Therefore, the perturbations on the sound field due to the presence of this solid can be neglected, thus p_1 can be approximated by

$$p_1 = P_B e^{-ikx} \tag{4}$$

where P_B is a real constant and k is the wave number. In this case, $p_1 e^{i\omega t}$ describes a travelling plane wave

NOMENCLATURE

c	sound speed in the fluid [m s^{-1}]
c_p	isobaric heat capacity per unit mass of the fluid [J kg^{-1} K^{-1}]
c_s	isobaric heat capacity per unit mass of the plate [J kg^{-1} K^{-1}]
k	wave number [m^{-1}]
K	thermal conductivity of the fluid [W K^{-1} m^{-1}]
l	semi-thickness of the plate [m]
p_1	space dependence of the acoustic pressure [Pa]
T_m	mean temperature [K]
T_1	space dependence of the temperature oscillation in the fluid [K]
u_1	space dependence of the velocity oscillation in the x direction [m s^{-1}]
$\dot{q}_{cy1}$	conductive heat flux in the y direction in the fluid [W m^{-2}]
$\dot{q}_{hx2}$	hydrodynamic heat flux in the x direction [W m^{-1}]
$\dot{Q}_{hx2}$	$= \int_0^\infty \dot{q}_{hx2}\,dy$ [W m^{-1}]
$\dot{w}_2$	acoustic power per unit volume generated or absorbed in the fluid [W m^{-3}]
$\dot{W}_2$	$= \int_0^\infty \dot{w}_2\,dy$ [W m^{-2}].

Greek symbols

β	thermal expansion coefficient [K^{-1}]
γ	ratio, isobaric to isochoric specific heats
δ_κ	fluid thermal penetration depth [m]
δ_s	plate thermal penetration depth [m]
δ_ν	fluid viscous penetration depth [m]
Δx	length of the plate in the x direction [m]
ν	kinematic viscosity of the fluid [m^2 s^{-1}]
κ	thermal diffusivity of the fluid [m^2 s^{-1}]
κ_s	thermal diffusivity of the plate [m^2 s^{-1}]
ρ_m	mean density of the fluid [kg m^{-3}]
ρ_s	mean density of the plate [kg m^{-3}]
σ	Prandtl number
ω	angular frequency [s^{-1}].

propagating in the x direction. The time average (over a time period) of the acoustic power per unit volume absorbed or generated is given by ref. [3], equation (30), $\overline{\dot{w}_2} = -(1/\rho_m)\overline{p(d\rho/dt)}$. Using the relation $d\rho = -\rho_m\beta\,dT + \gamma c^2\,dp$ (where γ is the ratio of specific heats of the fluid) and the equations (1) and (2) to eliminate u_1 and T_1, one obtains

$$\begin{aligned}\overline{\dot{w}_2} = {}& \frac{\beta}{2\rho_m\omega}\frac{1}{\sigma-1}\frac{1+\varepsilon_s\sqrt{\sigma}}{1+\varepsilon_s}\nabla T_m \\ &\times \operatorname{Im}\left(p_1^*\frac{dp_1}{dx}e^{-(1+i)y/\delta_\kappa}\right) \\ &+ \frac{T_m\beta^2\omega}{2\rho_m c_p}\frac{p_1 p_1^*}{1+\varepsilon_s}\operatorname{Im}(e^{-(1+i)y/\delta_\kappa}) \\ &- \frac{\beta}{2\rho_m\omega}\frac{1}{\sigma-1}\nabla T_m \operatorname{Im}\left(p_1^*\frac{dp_1}{dx}e^{-(1+i)y/\delta_\nu}\right), \end{aligned} \tag{5}$$

where p_1^* represents the complex conjugate of p_1. Integrating the result with respect to y from zero to infinity and substituting p_1 given by equation (4), we get

$$\overline{\dot{W}_2} = -\frac{\delta_\kappa}{4}\frac{\gamma-1}{\rho_m c^2}\frac{\omega P_B^2}{1+\varepsilon_s}\left(1 - \frac{1}{1+\sqrt{\sigma}}\frac{c_p k}{T_m\beta\omega^2}\nabla T_m\right). \tag{6}$$

The time average of the hydrodynamic heat flux along the x axis is given by ref. [3] (equation (26)) $\overline{\dot{q}_{hx2}} = \rho_m c_p\overline{u_1 T_1} - \beta T_m\overline{u_1 p_1}$. Using equations (1) and (2) to express this heat flux in terms of p_1, substituting its value from equation (4) and integrating the result with respect to y from zero to infinity, one obtains

$$\overline{\dot{Q}_{hx2}} = \frac{\delta_\kappa}{4}\frac{c_p}{\rho_m c^2}\frac{\gamma-1}{\beta\omega}\frac{1}{1+\varepsilon_s}\frac{1}{1+\sigma}kP_B^2 \left(1-(1-\sqrt{\sigma}) - \frac{c_p k}{T_m\beta\omega^2}\frac{\varepsilon_s\sigma+\sigma+\sqrt{\sigma}+1}{1+\sqrt{\sigma}}\nabla T_m\right). \tag{7}$$

From equation (6) it can be seen that $\overline{\dot{W}_2}$ is zero if ∇T_m is equal to

$$\nabla T_{crw} = \frac{T_m\beta\omega^2}{c_p k}(1+\sqrt{\sigma}). \tag{8}$$

In the same way, equation (7) gives $\dot{Q}_{hx2} = 0$ if ∇T_m is equal to

$$\nabla T_{crq} = -\frac{1-\sigma}{\varepsilon_s\sigma+\sigma+\sqrt{\sigma}+1}\frac{T_m\beta\omega^2}{c_p k}. \tag{9}$$

Note that $\overline{\dot{W}_2}$ is negative (acoustic energy is absorbed) if $\nabla T_m < \nabla T_{crw}$, and it is positive (energy is generated) if $\nabla T_m > \nabla T_{crw}$. Analogously, $\dot{Q}_{hx2}$ flows in the positive sense if $\nabla T_m > \nabla T_{crq}$ and in the opposite sense if $\nabla T_m < \nabla T_{crq}$. Thus, four regimes are possible (Fig. 1): (i) if $\nabla T_m > \nabla T_{crw}$, acoustic energy is generated using the heat flux from the hot region to the cold one (prime mover); (ii) for $0 < \nabla T_m < \nabla T_{crw}$, a noxious effect occurs because acoustic energy is absorbed to pump heat from the hot zone to the cold one, even though the heat flows in this sense naturally; (iii) if $\nabla T_{crq} < \nabla T_m < 0$, the acoustic energy is used to move heat in the same sense as the mean temperature gradient (refrigerator or heat pump); (iv) if $\nabla T_m < \nabla T_{crq}$, the 'noxious effect' appears again.

To pump heat from the cold region to the hot one

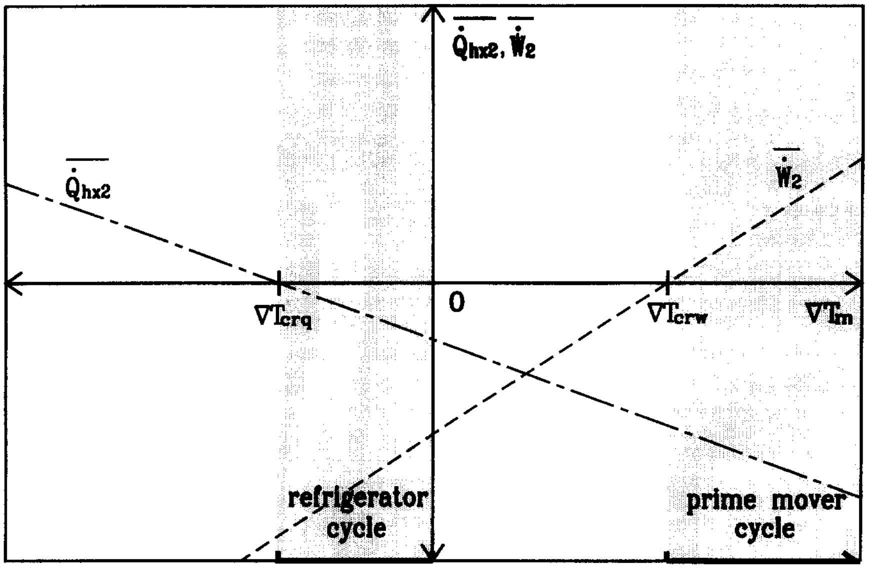

Fig. 1. $\overline{\dot{W}_2}$ and $\overline{\dot{Q}_{hx2}}$ vs ∇T_m; the intervals of values of ∇T_m where the thermoacoustic cycle corresponds to a refrigerator or a prime mover are shown.

the wave must propagate against the mean temperature gradient (as Ceperley [4, 5] concluded). As it can be seen, ∇T_{crq} is the highest value of ∇T_m that can exist in a refrigerator or a heat pump. If the value of σ decreases, the temperature difference between the cold and hot zones in a refrigerator or heat pump will increase, and the difference in temperature necessary to generate sound will be reduced. Therefore, the effect of the viscosity cannot be neglected (previous works [5, 6] have analyzed the thermoacoustic effect due to travelling waves in which the viscosity is zero or has been neglected).

From equation (5) it is observed that $\overline{\dot{w}_2}$ depends on the perpendicular distance to the plate. There are alternative regions where the acoustic energy is absorbed and others where it is generated. The width of those zones depends on the value of ∇T_m.

The conductive heat flux in the y direction in the fluid, produced by the temperature oscillations is $\dot{q}_{cy1} = -K(\partial t_1/\partial y)e^{i\omega t}$ (K is the thermal conductivity). The partial derivative with respect to y of the time average of the product $\dot{q}_{cy1}p_1e^{i\omega t}$ is given by

$$\frac{\partial}{\partial y}(\overline{\dot{q}_{cy1}p_1e^{i\omega t}})$$

$$= -\frac{K}{\delta_\kappa^2\rho_m\omega^2}\frac{1}{\sigma-1}\frac{1+\varepsilon_s\sqrt{\sigma}}{1+\varepsilon_s}\nabla T_m$$

$$\times \operatorname{Im}\left(p_1^*\frac{dp_1}{dx}e^{-(1+i)y/\delta_\kappa}\right)$$

$$-\frac{K}{\delta_\kappa^2\rho_m}\frac{T_m\beta}{c_p}\frac{p_1p_1^*}{1+\varepsilon_s}\operatorname{Im}(e^{-(1+i)y/\delta_\kappa})$$

$$+\frac{K}{\delta_\kappa^2\rho_m\omega^2}\frac{1}{\sigma-1}\nabla T_m\operatorname{Im}\left(p_1^*\frac{dp_1}{dx}e^{-(1+i)y/\delta_\nu}\right). \quad (10)$$

From equations (5) and (10), it is observed that the values of y where $\overline{\dot{q}_{cy1}p_1e^{i\omega t}}$ increases coincide with the absorption of acoustic power, owing to the fact that in these regions the net effect is a supply of heat in the semicycle when the fluid is expanded, and an extraction of heat in the semicycle when it is compressed. Similarly, at those regions where the acoustic power is generated, the net effect is that heat is added to the fluid in the semicycle of compression and it is removed in the one of expansion, in accordance with Rayleigh's hypothesis [7].

4. CONCLUSIONS

For each point x_0 on the plate there are two critical values of the mean temperature gradient; at the higher of these two values there is no absorbed (or generated) average acoustic energy on the $x = x_0$ plane, and at the lower the average hydrodynamic heat flux through the same plane is zero. It is shown that there is an interval of values of the mean temperature gradient at which the thermoacoustic cycle corresponds to a refrigerator cycle and another interval where the cycle corresponds to a prime mover. Outside these intervals there is a noxious effect owing to acoustic energy being absorbed to pump heat from the hot to the cold zone; a process that occurs naturally. It is also shown that the net acoustic energy generated or absorbed in a cycle depends on the distance to the plate and that the heat added or extracted to the fluid to generate or absorb acoustic energy is due to the conductive heat flux along a normal to the plate.

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